

NUMBER SEQUENCES

PATTERNS

TWO DIFFERENT CHAPTERS WITH
DIFFERENT TECHNIQUES.

ARITHMETIC SEQUENCE

4, 7, 10, 13, 16, 19

$$d = 13 - 10 = +3$$

20, 18, 16, 14, 12, 10

$$d = 10 - 12 = -2$$

First
Term

$$n\text{th term} = a + (n-1)d$$

→ common difference
 $d = \text{Next} - \text{previous}$.

↓
Term
number.

n tells us the position number

$n\text{th term}$ tells us value written on
that position

Q:

4, 7, 10, 13, 16, 19

$$d = N - P \\ = 19 - 16 = 3$$

- (i) Write down next two terms:
(ii) Find an expression, in terms of n , for $n\text{th term}$.

$$\begin{aligned} n\text{th term} &= a + (n-1)d \\ &= 4 + (n-1)(3) \\ &= 4 + 3n - 3 \end{aligned}$$

$$n\text{th term} = 3n + 1$$

- (iii) Find 50th term of this sequence.

$$n\text{th term} = 3n + 1$$

$$50^{\text{th}} \text{ term} = 3(50) + 1 = 151$$

(iv) Given that n^{th} term of the sequence is 61, find n .

$$n^{\text{th}} \text{ term} = 3n + 1$$

$$61 = 3n + 1$$

$$60 = 3n$$

$$n = 20$$

(v) Given that k^{th} term of the sequence is 91, find k .

$$n^{\text{th}} \text{ term} = 3n + 1$$

$$k^{\text{th}} \text{ term} = 3k + 1$$

$$91 = 3k + 1$$

$$90 = 3k$$

$$k = 30$$

IF THE SEQUENCE IS NOT ARITHMETIC,
THEN IT WILL BELONG TO ONE OF THESE
THREE FAMILIES

SQUARE

1

4

9

16

⋮

nth term = n^2

CUBE

1

8

27

64

⋮

n^3

TRIANGLE

$+2 \rightarrow 1$
 $+3 \rightarrow 3$
 $+4 \rightarrow 6$
 $+5 \rightarrow 10$
 $+6 \rightarrow 15$
 $+7 \rightarrow 21$

○

○○

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THESE nth terms only work if these sequences
start from 1.

IF THEY ARE NOT STARTING FROM 1:

SHIFTS

1. Forward shift: Number starts after skipping
first term(s)

So we replace $n \rightarrow n+1$ etc

2. Backward shift: Number starts before
first term i.e 0.

So we replace $n \rightarrow n-1$

Q. Write down n th term of this sequence.

4, 9, 16, 25

These are square numbers. But these are shifted.

ORIGINAL SQUARE: 1 4 9 16 n^2

FORWARD SHIFT 4 9 16 $(n+1)^2$

$n \rightarrow n+1$

Ans. $(n+1)^2$

Q. Write down n th term of this sequence.

8, 27, 64, 125

These are cube numbers but shifted.

ORIGINAL CUBE: 1 8 27 64 n^3

SHIFT FORWARD 8 27 64 $(n+1)^3$

$n \rightarrow n+1$

Ans. $(n+1)^3$

Q. Write down n th term of this sequence.

27, 64, 125

These are cube numbers but shifted.

ORIGINAL CUBE: 1 8 27 64 n^3

FORWARD SHIFT 27 64 $(n+2)^3$

$n \rightarrow n+2$

Ans. $(n+2)^3$

Q. write down n th term of this sequence.

0, 1, 8, 27, 64, ...

These are cube numbers but shifted.

ORIGINAL CUBE: 1 8 27 64 ... n^3

BACKWARD SHIFT: 0 1 8 27 64 - . . . (n-1)³

$$n \rightarrow n-1$$

Ans: $(n-1)^3$

Q. write down n th term of this sequence.

0, 1, 4, 9, 16, 25, ...

These are square numbers. But these are shifted.

ORIGINAL SQUARE: 1 4 9 16 n^2

Backward Shift: 0 1 4 9 16 . . . $(n-1)^2$

$$n \rightarrow n-1$$

Ans: $(n-1)^2$

Q Write down n th term of this sequence.

3, 6, 10, 15, ...

These are triangle numbers. Shifted.

ORIGINAL TRIANGLE:

1	3	6	10	...	$\frac{1}{2}n(n+1)$
					$\downarrow \quad \downarrow$
					$n \quad n+1$

SHIFT FORWARD

3	6	10	...	$\frac{1}{2}(n+1)(n+2)$
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$$n \rightarrow n+1$$

Q Write down n th term of this sequence.

0, 1, 3, 6, 10, 15, ...

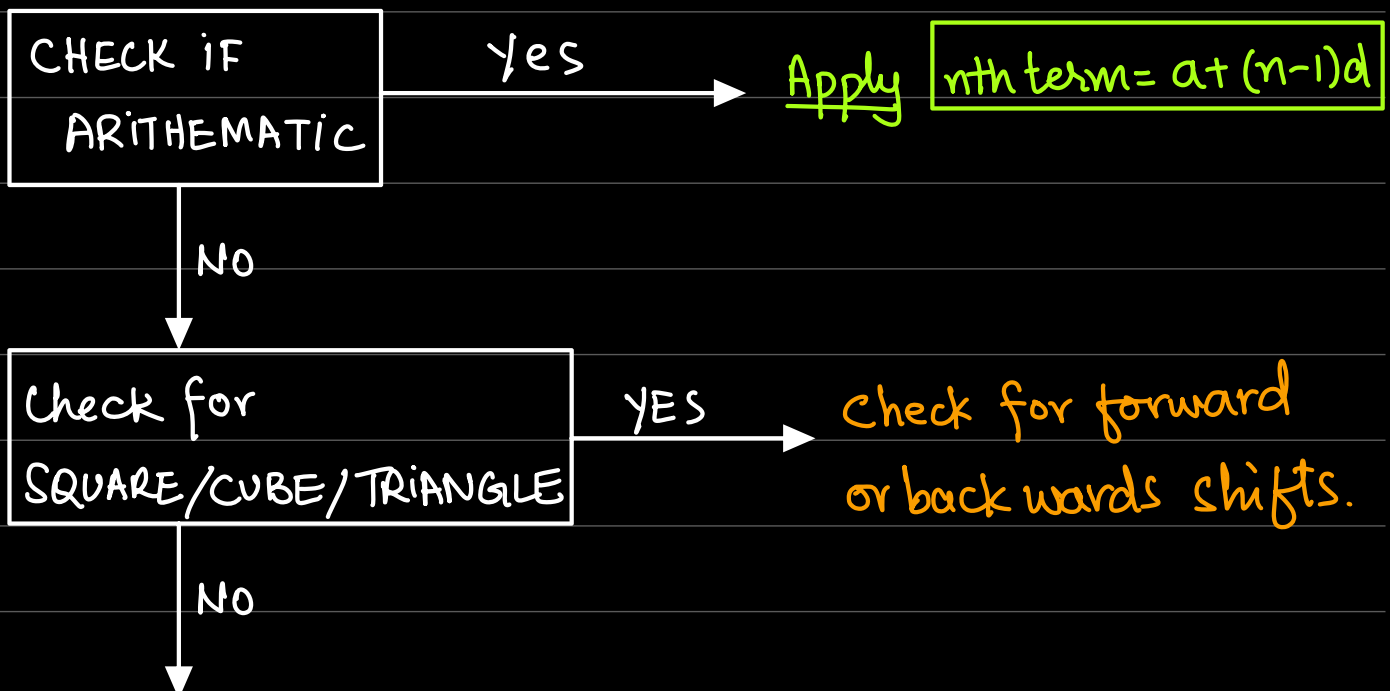
These are triangle numbers. Shifted.

ORIGINAL TRIANGLE: 1 3 6 10 ... $\frac{1}{2}n(n+1)$

BACK SHIFT: 0 1 3 6 10 $\frac{1}{2}(n-1)(n)$ ✓
 $n \rightarrow n-1$

$$\frac{1}{2} n(n-1) \checkmark$$

EXAM TECHNIQUE



Now make a new table and include all the sequences of question in the table. First row will be term number.

Term	1	2	3	n
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(a)				
(b)				
(c)				

NOW LOOK FOR VERTICAL PATTERN.

Q:

Term	1	2	3	4	5	6						n
A	4	7	10	13	16	19	Arithmetic:					$3n+1$
B	1	4	9	16	25	36	SQUARE (NO SHIFT)					n^2
C	8	27	64	125	216	343	CUBE: ($n \rightarrow n+1$)					$(n+1)^3$
D	0	2	6	12	20	30	D = B - Term					$n^2 - n$
E	9	31	73	141	241	379	E = B + C					$n^2 + (n+1)^3$
F	8	54	192	500	1080	2058	F = Term $\times$ C					$n(n+1)^3$
G	2	8	18	32	50	72	G = 2 $\times$ B					$2n^2$

- (i) Complete the table for Term 5 and 6 .
(ii) Complete the nth term column for each.

A 4, 7, 10, 13....

ARITHMETIC

$$\begin{aligned}
 \text{nth term} &= a + (n-1)d \\
 &= 4 + (n-1)(3) \\
 &= 4 + 3n - 3
 \end{aligned}$$

$$= 3n + 1$$

- 13 The first four terms of a sequence are 55, 53, 49, 41.
The n th term of this sequence is $57 - 2^n$.

(a) Calculate the fifth term.

$$n\text{th term} = 57 - 2^n$$

$$\begin{aligned} 5^{\text{th}} \text{ term} &= 57 - 2^5 \\ &= 57 - 32 \\ &= 25 \end{aligned}$$

Answer 25 [1]

(b) Write down the n th term of the sequence 56, 55, 52, 45 ...

Term	1	2	3	4		n
(a)	55	53	49	41		$57 - 2^n$
(b)	56	55	52	45		$n + 57 - 2^n$

Answer $n + 57 - 2^n$ [1]

PATTERNS

LOOK OUT FOR THINGS THAT ARE NOT CHANGING. THOSE ARE PATTERN.
REST IS SEQUENCES.

23 A pattern of numbers is given below.

Row 1 $\rightarrow \frac{1}{1 \times 2} = \frac{1}{1} - \frac{1}{2}$

Row 2 $\rightarrow \frac{1}{2 \times 3} = \frac{1}{2} - \frac{1}{3}$

Row 3 $\rightarrow \frac{1}{3 \times 4} = \frac{1}{3} - \frac{1}{4}$

Row 4 $\rightarrow \frac{1}{4 \times 5} = \frac{1}{4} - \frac{1}{5}$

(a) Write down Row 10.

Answer $\frac{1}{10 \times 11} = \frac{1}{10} - \frac{1}{11}$ [1]

(b) Adding the first two rows gives the result $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} = \frac{1}{1} - \frac{1}{3} = \frac{2}{3}$ $\rightarrow +1$

Adding the first three rows gives the result $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} = \frac{1}{1} - \frac{1}{4} = \frac{3}{4}$ $\rightarrow +1$

(i) Write down the result of adding the first four rows.

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} = \frac{1}{1} - \frac{1}{5} = \frac{4}{5}$$

Answer [1]

(ii) Use the pattern to write down

(a) the value of $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{19 \times 20} = \frac{19}{20}$

Answer [1]

(b) the number of rows that add up to $\frac{109}{110}$, $\rightarrow$ rows

Answer 109 rows. [1]

(c) an expression, in terms of n , for the result of adding the first n rows. $= \frac{n}{n+1}$

Answer $\frac{n}{n+1}$ [1]

10 Look at this pattern.

Line 1: $2^2 - 0^2 = 4 \times 1$

Line 2: $3^2 - 1^2 = 4 \times 2$

Line 3: $4^2 - 2^2 = 4 \times 3$

Line 4: $5^2 - 3^2 = 4 \times 4$
 $\square^2 - \square^2 = 4 \times \square$

(a) Write down the 7th line of the pattern.

Answer (a) $8^2 - 6^2 = 4 \times 7$

(b) Write down the n th line of the pattern.

Answer (b) $(n+1)^2 - (n-1)^2 = 4 \times n$ [1]

(c) Use the pattern to find $521^2 - 519^2$.

$(n+1)^2 - (n-1)^2 = 4 \times n$
 $521^2 - 519^2 = ?$
 $n+1=521$ $n-1=519$
 $n=520$ $n=520$
 $? = 4 \times n$
 $= 4 \times 520$
 $= 2080$

Answer (c) 2080 [1]

(d) Use the pattern to find the positive integers x and y such that $x^2 - y^2 = 484$.

$(n+1)^2 - (n-1)^2 = 4 \times n$
 $x^2 - y^2 = 484$
 $x = n+1$ $y = n-1$ $4 \times n = 484$
 $n = \frac{484}{4} = 121$
 $x = 121+1$ $y = 121-1$
 $x = 122$ $y = 120$

Answer (d) $x = 122$
 $y = 120$ [1]

2, 3, 4, 5... arithmetic
 n th term $= a + (n-1)d$
 $= 2 + (n-1)(1)$
 $= 2 + n - 1$
 $= n + 1$

0, 1, 2, 3... arithmetic
 n th term $= a + (n-1)d$
 $= 0 + (n-1)(1)$
 $= n - 1$

1, 2, 3, 4... arithmetic
 n th term $= 1 + (n-1)(1)$
 $= 1 + n - 1$
 $= n$ [1]

19 The first four lines of a pattern of numbers are shown below.

$$\text{1st line} \quad 3^2 - 1^2 = 8 \times 1$$

$$\text{2nd line} \quad 5^2 - 1^2 = 8 \times (1 + 2)$$

$$\text{3rd line} \quad 7^2 - 1^2 = 8 \times (1 + 2 + 3)$$

$$\text{4th line} \quad 9^2 - 1^2 = 8 \times (1 + 2 + 3 + 4)$$

(a) Write down the 7th line of the pattern.

Answer [1]

(b) Write down an expression, in terms of n , to complete the n th line of the pattern.

Answer $= 8 \times (1 + 2 + 3 + 4 + \dots + n)$ [1]

(c) Using the n th line of the pattern, show that $1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2}$.

[2]